A Statistical Algorithm for Estimating Speed from Single Loop Volume and Occupancy Measurements *

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Abstract

This paper presents an algorithm for estimating mean traffic speed using volume and occupancy data from a single inductance loop. The algorithm is based on the statistics of the measurements obtained from a traffic management system. The algorithm produces an estimate of speed and provides a reliability test for the speed estimate.

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INTRODUCTION

In this paper we present an algorithm for estimating mean traffic speed using single inductance loop measurements of volume (counts of vehicle over a duration) and occupancy (the fraction of some total duration during which the inductance loop senses the presence of a vehicle). Mechanisms to estimate speed from single loops has been of interest to traffic engineers for some time as speed is not directly observable from single loop measurements. (Hall and Persaud, 1988; Leutzbach, 1988; Persaud and Hurdle, 1988; Hall and Gunter, 1986; Persaud and Hall, 1989; Hall, 1987; Dillon and Hall, 1987; Gunter and Hall, 1986; Dailey, 1992). Recent advanced traveler information system (ATIS) initiatives have created a requirement for a robust solution to this problem for a new class of applications, namely those which provide information to travelers. It is as a result of such an initiative (Seattle Wide Area Information for Travelers, SWIFT) that the present algorithm was formulated.

In this paper we acknowledge the statistical nature of the measurements made using inductance loops and present an algorithm to estimate speed that accounts for the statistical nature of the estimate as well as providing a reliability test for our estimate. There are four measurements that are made by a traffic management system, Volume $N(t)$, Occupancy $O(t)$, speed $s(t)$, and vehicle length $l(t)$ (but only volume and occupancy are available from single loops). These measurements are by their nature realizations taken from the probability distributions of the underlying variables, at the time the measurement are made. Observations of these variables are typically combined to create estimates of speed; for example, several authors have used a ratio of volume ($\hat{n}$) and occupancy ($\hat{o}$) with correction (g) to estimate speed $\hat{s} = \hat{n}/g\hat{o}$. (Hall and Persaud, 1988; Leutzbach, 1988; Wardrop, 1952; Persaud and Hurdle, 1988; Kurkjian et al., 1980; Nahi, 1973; Payne et al., 1987) ATIS efforts typically require estimates of speed and travel times but rely almost completely on the measurements made by traffic management systems, and as such, require the use of single inductance loop speed estimates.

Previous work has not explicitly included the statistics of the estimated quantities when making estimates of variables that are not observable. This work explicitly considers the statistics of estimates created using observations from traffic management systems. The typical measurements are volume (N_i) and occupancy (O_i), and the relationship between volume, occupancy, speed s_{ij}, and length of the jth vehicle l_{ij} is

$$O_i = \frac{1}{T} \sum_{j=1}^{N_i} \frac{l_{ij}}{s_{ij}}, \tag{1}$$

where T is the duration of the measurement. The speed and vehicle length are random variables with mean values and statistical distributions. We can express this by writing the speed and length observations as the expected value (mean) and some deviation $(\Delta l_{ij}, \Delta s_{ij})$ that occurs for this observation,

$$l_{ij} = \bar{l} + \Delta l_{ij} \tag{2}$$

$$s_{ij} = \bar{s} + \Delta s_{ij}. \tag{3}$$

Combining these terms in the form of the RHS of equation (1), we get

$$\frac{l_{ij}}{s_{ij}} = \frac{\bar{l}}{\bar{s} + \Delta s_{ij}} + \frac{\Delta l_{ij}}{\bar{s} + \Delta s_{ij}}, \tag{4}$$

where the statistics of the deviation term are selected such that $E\{\Delta l_{ij}\} = E\{\Delta s_{ij}\} = 0$ and $E\{*\}$ is the expected value operator.

Each measurement produces a pair of volume (N_i) and occupancy values (O_i). To use the statistics of these measurements, let E_i denote the conditional expectation over all realizations that have the volume N_i. The conditional expected value of equation (1) is

$$E_i\{O_i\} = \frac{N_i}{T} E_i \left\{ \frac{l_{ij}}{s_{ij}} \right\}. \tag{5}$$

Inserting equation (4) in (5), we get

$$E_i \left\{ \frac{l_{ij}}{s_{ij}} \right\} = E_i \left\{ \frac{\bar{l}}{\bar{s} + \Delta s_{ij}} + \frac{\Delta l_{ij}}{\bar{s} + \Delta s_{ij}} \right\}. \tag{6}$$

Rearranging the RHS, assuming that the variables $(\frac{1}{\Delta s_{ij}})$ and Δl_{ij} are independent, (we note that the correlation coefficient (Bendat and Piersol, 1986) between $(\frac{1}{\Delta s_{ij}})$ and Δl_{ij}, $\rho_{\Delta l_{ij}(\frac{1}{\Delta s_{ij}})} = 0.018$ for the data presented later in this paper; this low value for the correlation coefficient does not prove independence but does suggest that such an assumption is reasonable) and recognizing that $E\{\Delta l_{ij}\} = 0$, we get

$$E_i\left\{\frac{l_{ij}}{s_{ij}}\right\} = E_i\left\{\frac{\bar{l}}{\bar{s}+\Delta s_{ij}}\right\} = \frac{\bar{l}}{\bar{s}}E_i\left\{\frac{1}{1+\dfrac{\Delta s_{ij}}{\bar{s}}}\right\}. \tag{7}$$

Expanding the RHS in a power series results in

$$E_i\left\{\frac{l_{ij}}{s_{ij}}\right\} = \frac{\bar{l}}{\bar{s}}E_i\left\{1-\frac{\Delta s_{ij}}{\bar{s}}+\frac{\Delta s_{ij}^2}{\bar{s}^2}-\frac{\Delta s_{ij}^3}{\bar{s}^3}+\ldots\right\}. \tag{8}$$

Noting that $E\{\Delta s_{ij}\} = 0$, approximating the series with three terms, and inserting the result in equation (5) results in

$$E_i\left\{O_i\right\} = \frac{N_i}{T}\frac{\bar{l}}{\bar{s}}\left[1+\frac{E_i\{\Delta s_{ij}^2\}}{\bar{s}^2}\right]. \tag{9}$$

The variance of the speed estimate can be written, $\sigma_s^2 = E_i\{\Delta s_{ij}^2\}$, and by substituting and rearranging, we get

$$N_i = \frac{\bar{s}T}{\bar{l}}E_i\left\{O_i\right\}\left[\frac{\bar{s}^2}{\sigma_s^2+\bar{s}^2}\right]. \tag{10}$$

The measurement of the occupancy is also a random variable with some mean and some deviation from that mean for the ith measurement; we can express this as

$$O_i = \bar{O}+\Delta O_i \qquad \bar{O} = E_i\left\{O_i\right\}. \tag{11}$$

Substituting (11) into (10), we arrive at

$$\frac{N_i}{O_i} = \frac{\bar{s}T}{\bar{l}}\left[\frac{\bar{s}^2}{\sigma_s^2+\bar{s}^2}\right] - \frac{\Delta O_i}{O_i}\frac{\bar{s}T}{\bar{l}}\left[\frac{\bar{s}^2}{\sigma_s^2+\bar{s}^2}\right]. \tag{12}$$

This form has a deterministic component containing only moments of the speed distribution and a stochastic component containing ΔO_i. We consider the solution of the deterministic component in the next section.

DETERMINISTIC MEASURMENTS

In the case where there are perfect measurements (e.g. $\Delta O_i = 0$) and each realization of volume and occupancy is equal to the mean of the probability distribution for that measurement,

$$\frac{N_i}{O_i} = \frac{\bar{s}T}{\bar{l}} \left[\frac{\bar{s}^2}{\sigma_s^2 + \bar{s}^2} \right]. \tag{13}$$

Previous authors have asserted that a ratio of measured volumes and occupancies converted to density by a constant can be used to estimate speed. (Hall and Persaud, 1988; Persaud and Hurdle, 1988; Hall and Gunter, 1986; Ross, 1988) Rearranging equation (13) to the form used by these pervious authors,

$$\frac{N_i}{O_i} \left(\frac{\bar{l}}{T} \right) \left[\frac{\sigma_s^2}{\bar{s}^2} + 1 \right] = \bar{s}, \tag{14}$$

demonstrates that an estimate which does not consider the variability of the speed (contained in σ_s^2) has a bias. An estimate based on error-free measurements can be obtained by solving

$$O_i \frac{T}{\bar{l}} \bar{s}^3 - N_i \bar{s}^2 - N_i \sigma_s^2 = 0 \tag{15}$$

for $\bar{s}$. Equation (15) has the form $f(s) = 0$ and can be solved for the real root[1]. This "root finding" solution provides an unbiased estimator for $\bar{s}$ when there are idealized noiseless measurements; however, such is never the case. The next section provides an algorithm that addresses real measurements.

STOCHASTIC MEASURMENTS

Measurements from a traffic management system are realizations from statistical distributions. To address the variability of the observations, we present a filtering approach.

[1] The formula of DeMoivre allows for one real and two imaginary roots. (Kreyszig, 1979)

The general form for the dynamics and observer equations for a Kalman filter are (Bozic, 1984)

$$X_k = g_k(X_{k-1}) + w_{k-1} \tag{16}$$

$$Z_k = h_k(X_k) + v_k. \tag{17}$$

For the kth time step, we select our state variables to be the estimate of speed for the last two time steps. This autoregressive-like approach explicitly identifies a temporal correlation between speed estimates and recognizes that $\bar{s}$ has some inherent variation in addition to the noise component. For our observables, we use the ratio of the measurements for the two previous time steps. The selection of $\frac{O_k}{N_k}$ for our observable is based on examining equation (1) and noting that the variable O_i is inversely proportional to the state variable $\bar{s}$. The number of observations (N_i) used to construct O_i is used to normalize the observed value of O_i to a per vehicle basis. Further, when $N_i = 0$, the observation from that time step is undefined (as opposed to having zero value). We also note that in equation (12) there are deterministic and stochastic components. We use the deterministic portion to construct the measurement function ($h_k(\mathbf{X}_k)$, and we use

$$\mathbf{X} = \begin{bmatrix} \bar{s}_{k-1} \\ \\ \bar{s}_{k-2} \end{bmatrix}, \qquad \mathbf{Z} = \begin{bmatrix} \dfrac{O_k}{N_k} \\ \\ \dfrac{O_{k-1}}{N_{k-1}} \end{bmatrix}, \qquad h_k(\mathbf{X}_k) = \frac{\bar{l}}{T} \begin{bmatrix} \dfrac{\sigma_s^2 + \bar{s}_{k-1}^2}{\bar{s}_{k-1}^3} \\ \\ \dfrac{\sigma_s^2 + \bar{s}_{k-2}^2}{\bar{s}_{k-2}^3} \end{bmatrix} \tag{18}$$

where the measurement equation for $h_k(\mathbf{X}_k)$ is nonlinear in the state variables. The linear Kalman filter equations are written (Bozic, 1984)

$$\mathbf{X}_k = \mathbf{G}_k \mathbf{X}_{k-1} + \mathbf{w}_{k-1} \tag{19}$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{v}_k, \tag{20}$$

where the measurement equation is a linear function of the state variables. To use the linear filtering result, we adopt the extended Kalman filter approach which linearizes the measurement equation from (17) about a point $\mathbf{X}_k^p$ (for implementation we select this point to be the last $\mathbf{X}_k$),

$$h_k(\mathbf{X}_k) = h_k(\mathbf{X}_k^p) + dh(\mathbf{X}_k^p)(\mathbf{X}_k - \mathbf{X}_k^p), \tag{21}$$

and create a new measurement equation,

$$\hat{\mathbf{Z}}_k = \hat{\mathbf{H}}_k \mathbf{X}_k + \mathbf{v}_k, \tag{22}$$

where

$$\hat{\mathbf{Z}}_k = \{\mathbf{Z}_k - h_k(\mathbf{X}_k^p) + dh(\mathbf{X}_k^p)\mathbf{X}_k^p\} \qquad \hat{\mathbf{H}}_k = dh(\mathbf{X}_k^p), \tag{23}$$

and

$$dh_k(\mathbf{X}_k) = \begin{bmatrix} -\frac{3\bar{l}}{T}\left[\frac{\bar{s}_{k-1}^2 + \sigma_s^2}{\bar{s}_{k-1}^4}\right] & 0 \\ 0 & -\frac{3\bar{l}}{T}\left[\frac{\bar{s}_{k-2}^2 + \sigma_s^2}{\bar{s}_{k-2}^4}\right] \end{bmatrix}. \tag{24}$$

Our state-transition matrix $\mathbf{G}$ provides weights for the contribution of $\bar{s}$ from the previous two time steps,

$$\mathbf{G} = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix}, \tag{25}$$

where a and b are selected using forward/backward least squares estimates of the AR(2) coefficients for the experimentally measured speed. (Percival and Walden, 1993) The noise contributions are

$$\mathbf{Q}_k = E\left\{\mathbf{w}_k \mathbf{w}_k^T\right\} \qquad \mathbf{R}_k = E\left\{\mathbf{v}_k \mathbf{v}_k^T\right\} \tag{26}$$

where

$$\mathbf{Q} = \begin{bmatrix} \sigma_s^2 & 0 \\ 0 & \sigma_s^2 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \sigma_{\frac{O}{N}}^2 & 0 \\ 0 & \sigma_{\frac{O}{N}}^2 \end{bmatrix}. \tag{27}$$

With these definitions, we can use the linear filter solution

$$\mathbf{P}_k^1 = \mathbf{G}\mathbf{P}_{k-1}\mathbf{G}^T + \mathbf{Q}_{k-1} \tag{28}$$

$$\mathbf{K}_k = \mathbf{P}_k^1 \hat{\mathbf{H}}_k^T \left[\hat{\mathbf{H}}_k \mathbf{P}_k^1 \hat{\mathbf{H}}_k^T + \mathbf{R}_k\right]^{-1} \tag{29}$$

$$\mathbf{P}_k = \mathbf{P}_k^1 - \mathbf{K}_k \hat{\mathbf{H}}_k \mathbf{P}_\mathbf{k}^\mathbf{1} \tag{30}$$

$$\mathbf{X}_k = \mathbf{G}\mathbf{X}_{k-1} + \mathbf{K}_k \left[\hat{\mathbf{Z}}_k - \hat{\mathbf{H}}_k \mathbf{G}\mathbf{X}_{k-1}\right] \tag{31}$$

from reference (Bozic, 1984) to update the state variables at each time step. This provides an algorithm to create the best single step estimate of the state variables that maximize the likelihood function given in equation (1) of reference (Bell, 1994). The confidence we place in this estimate can be tested by calculating the mean length for each estimate using

$$\bar{l}_i = \frac{O_i T}{N_i}\left[\frac{\bar{s}_k^3}{\sigma_s^2 + \bar{s}_k^2}\right] \tag{32}$$

and comparing this estimate with long time estimates of the mean ($\bar{l}$) and standard deviation ($\sigma_{\bar{l}}$) of the length distribution. If $(\bar{l} - c) < \bar{l}_k < (\bar{l} + d)$ (where c and d are selected based on the statistics of $\bar{l}$), the speed estimate is deemed to be acceptable.

EMPIRICAL RESULTS

In this section we present empirical results for the two estimators presented and compare these results with empirical speed trap measurements. The two new estimators presented here are: (1) the "root finding" method based on the assumptions of deterministic values and (2) the filtering method. The results are compared quantitatively over a range of traffic conditions.

Measurements of traffic on Interstate 5 in Seattle were taken from the WSDOT Traffic Management System (TMS). The sites selected for testing have pairs of loops that act as a speed trap as well as making individual loop estimates of volume and occupancy. The loops detector stations average (sum) the values for volume and occupancy over a twenty-second interval, and all the data presented here are for twenty-second averages.

In the algorithms presented here, a mean value for length $\bar{l}$ is necessary, as well as an estimate of the variability of the speed σ_s. To obtain a mean length for the calculation, we used the empirical length estimates from the TMS over a six-day period. The histogram of the observed lengths is shown in Figure 1, and the mean value used to seed the calculation is 25.63 feet. This empirically generated distribution of lengths is also useful for testing the acceptability of the filter estimate. This test is described later in this paper.

The first empirical result presented here is the speed estimate from the roots of equation (15). These speed estimates are unbiased point estimates of the speed given σ_s and $\bar{l}$. A comparison of the root speed estimate and the speed measurement from the speed trap is shown in Figure 2. The estimate has a larger variance than the measured data but generally follows the character of the measured speed. The moments of the deviation of the estimates of speed from the observed twenty-second average speed (e.g. $\mu_e = E\{(s - \hat{s}_e)\}$) are indicative of the bias in the estimator. The estimates using a "g" factor (taken from the TMS) have a bias relative to the measurements ($\mu_g = 3.1$). The root methodology estimate has little bias relative to the measurements ($\mu_r = 0.07$).

The second speed estimator, the Kalman filter approach, is derived from equations (28) through (31). The estimate is plotted with the empirical speed from the speed trap associated with the loop detector from which we obtain the volume and occupancy. In this case, the estimate reflects the variability in the speed as a function of time with a smaller variance than the measured speed. It is important to note that the speed trap realization is a point estimate of the traffic conditions and is not the mean value of the speed distribution for the traffic conditions as they exist. The reliability of the estimate of speed can be addressed using the knowledge of the statistics for mean length as embodied in Figure 1 and a calculation of $\bar{l}_i$ from equation (32). Speed estimates that produce $\bar{l}_i$ values that are sufficiently far from the probability mass of the distribution are less reliable than those near the most probable lengths. The empirical distribution of length is an asymmetric, strongly peaked distribution containing 95% of the probability mass in the range of 15-40 feet and with small probability of occurrence (less than 0.008) outside this range. The selection of the criteria for accepting the validity of a speed estimate is an engineering judgment based on the probability of occurrence. We define acceptable estimates of speed to be those estimates that produce a length estimate (for a twenty-second average length) in the range of 15-40 feet, and those outside this range are deemed unreliable. This criteria provides an independent means to evaluate the reliability of our speed estimates. Figure 4 presents the mean lengths produced using the filter estimates for speed. For comparison, Figure 5 presents the lengths as measured by the TMS. It is clear that in some cases the estimate made by the filter violates the acceptability criteria and would not be used for subsequent modeling calculations and traveler information systems. The ability to identify estimates that are not acceptable sets this methodology apart from previous work.

CONCLUSION

This paper presents an algorithm to estimate speed from single inductance loops, as well as providing an acceptability test for the estimates. The algorithm specifically acknowledges the statistics of the problem, and the acceptability test uses the statistics of one of the observables to set criteria for evaluating the reliability of the estimate. The algorithm, when converted to algebraic form, is appropriate for use with single inductance loop data in both traffic management systems and traveler information systems.

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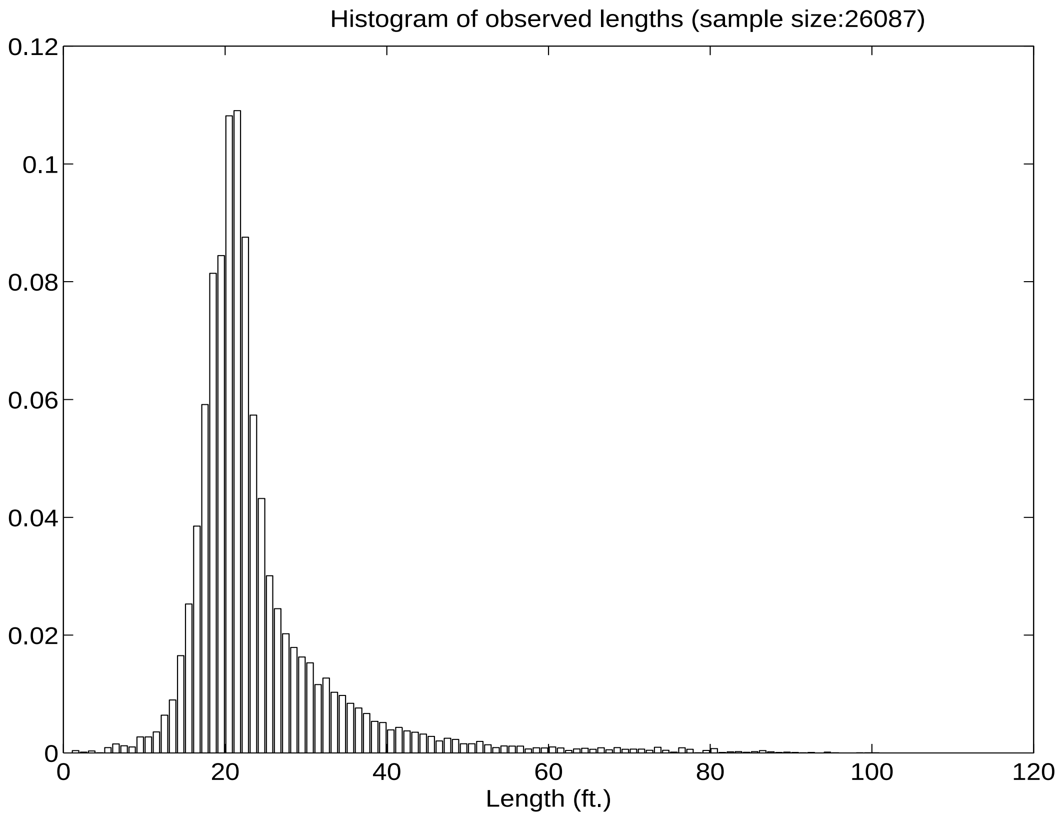

Figure 1: Car length histogram.

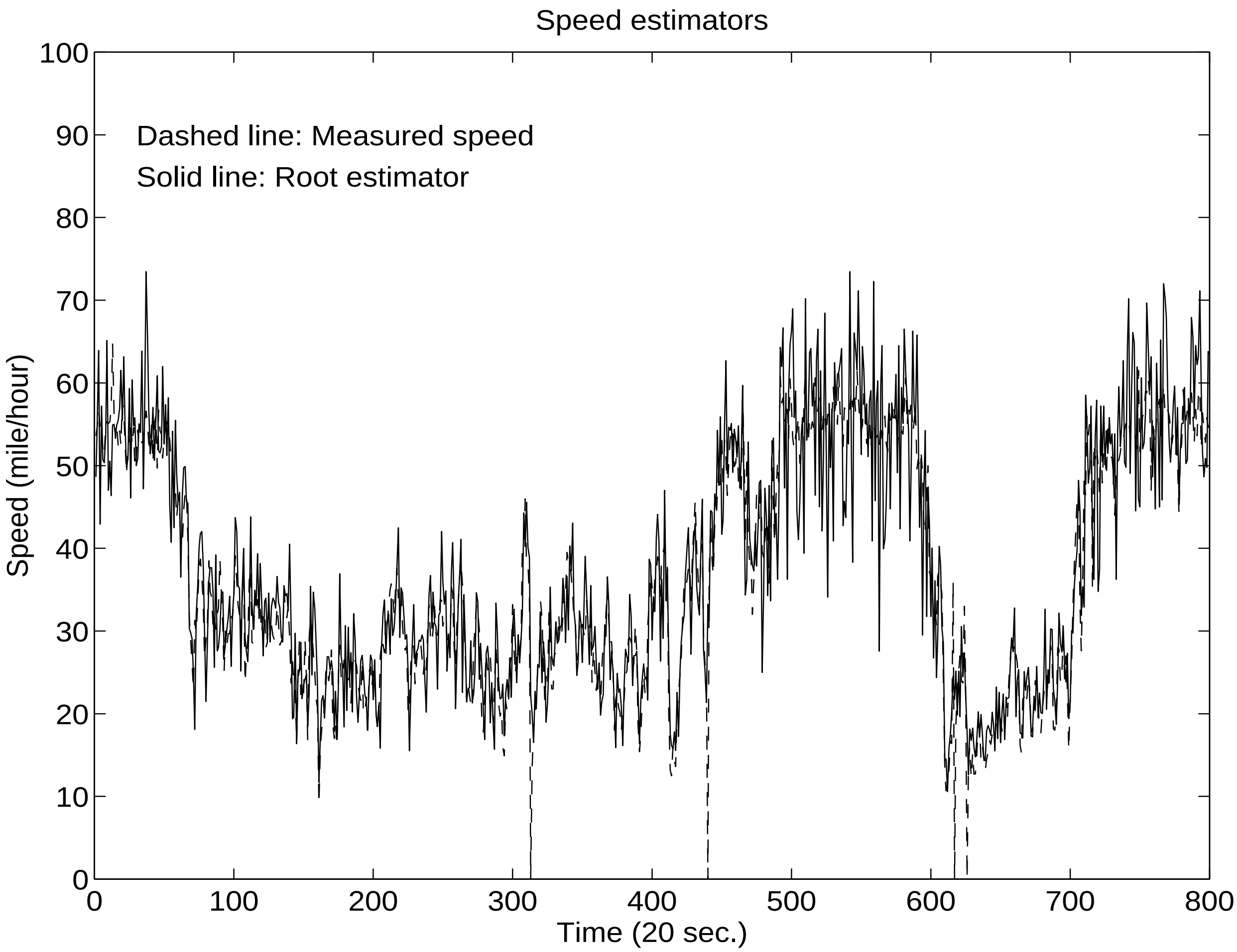

Figure 2: Speed estimates and observations.

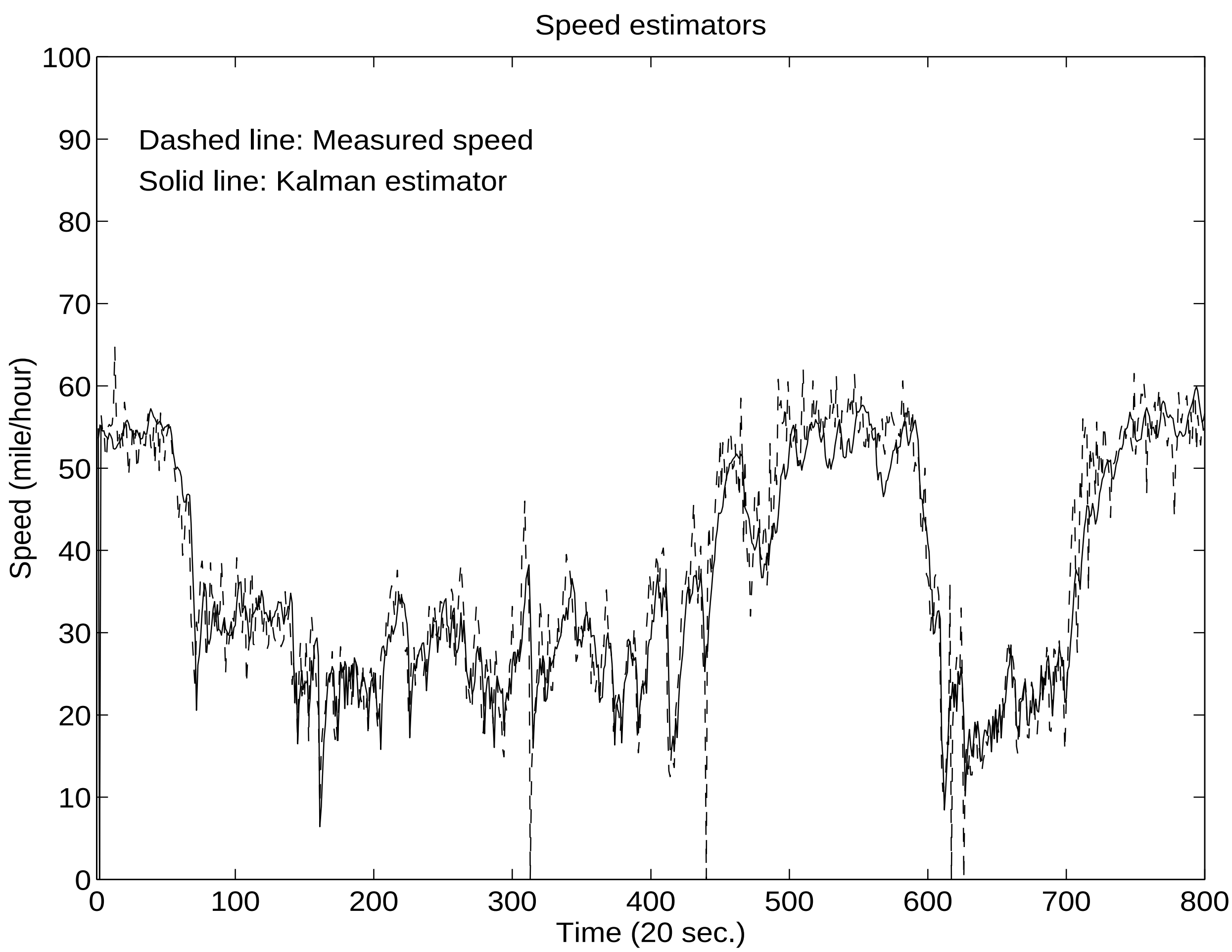

Figure 3: Speed estimates and observations.

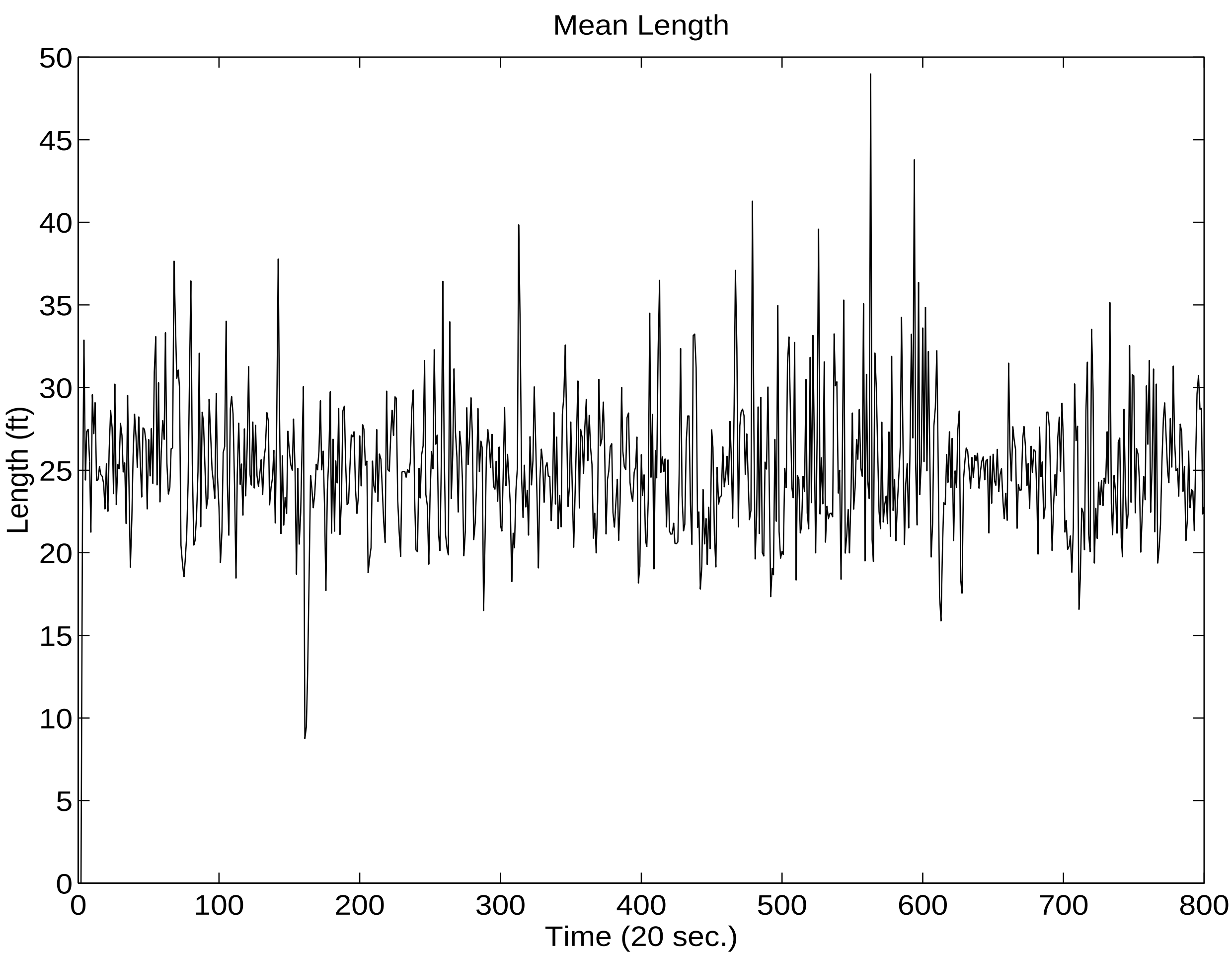

Figure 4: Car length estimates as a function of time.

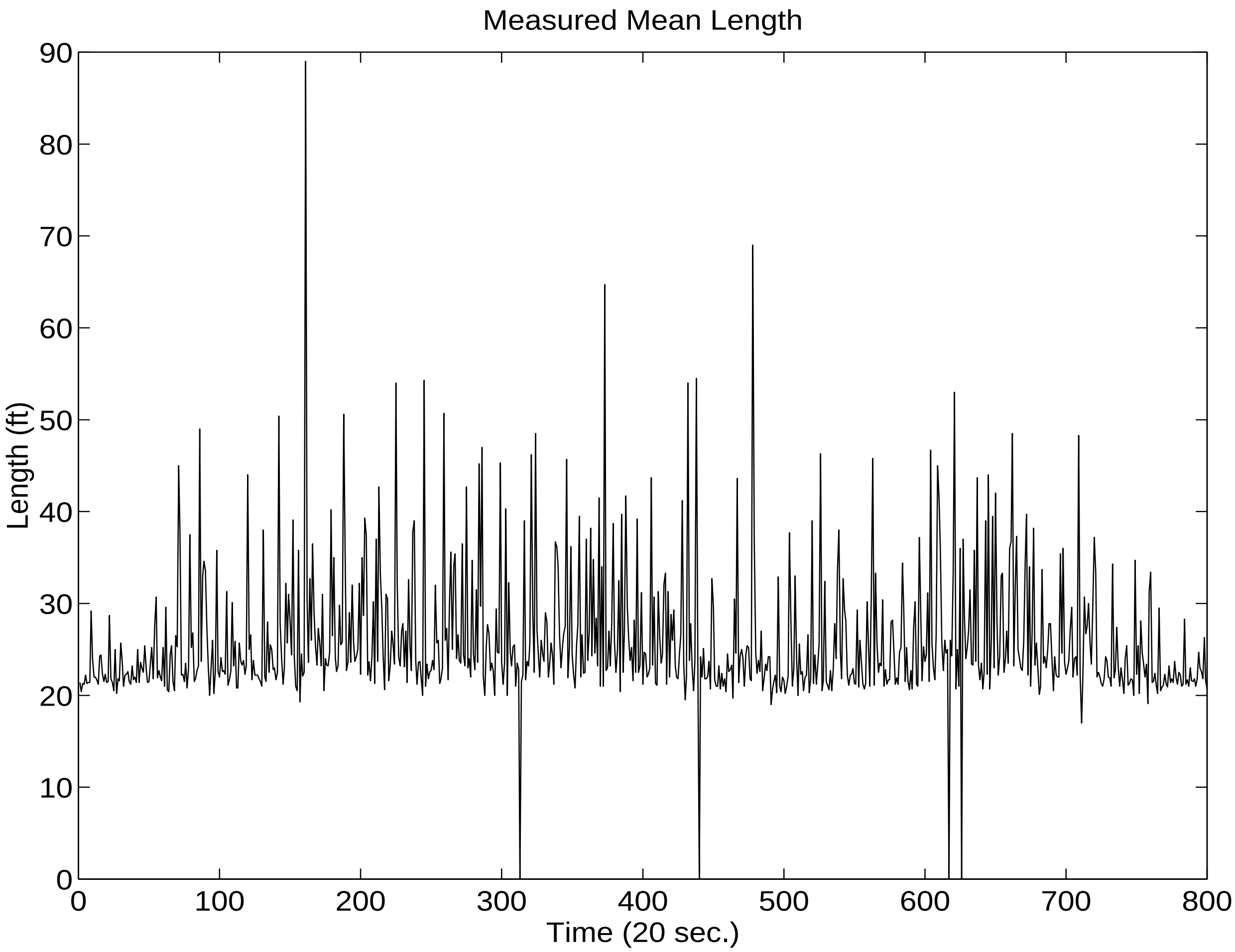

Figure 5: Car length as measured by the TMS.